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Two friendly rules for the turbulent heat transfer enhancement

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Abstract

The effect of roughness elements on heat transfer in fluids is different from their effect in gases because of the different thermal resistance of the viscous sublayer. The distinctive feature of the process was examined in every detail and generalised by two rules deduced, one for fluids with $Pr > 5$ and another for gases. For fluids, small equally dispersed roughness elements can disturb the viscous sublayer sufficiently. For gases, the enhancement is reached by creation of many reattachment zones after obstacles. Two- to three-fold heat transfer enhancement within the limits of Reynolds analogy $2St/c_{\rm f}$ is attainable.

Keywords: Heat transfer; Roughness; Turbulent

1. Introduction

Convective heat transfer intensification has been widely investigated in the past. In addition to all this well-documented and sufficiently generalised material in handbooks, own investigation results concerning small height roughness elements are presented in this work. It follows from the main heat exchange law that the whole content of transfer is embraced by the heat transfer coefficient. Augmentation of the heat transfer coefficient is closely related to the control of the flow eddies structure. Turbulisation by eddies caused by the roughness elements is different in fluids and gases, because in the fluids with $Pr > 1$ the thermal resistance concentrates itself close to the wall, while in the gases it is spread over the whole thickness of the boundary layer. Low density of air and other gases while allowing the velocity to increase, prevents the flow from taking more heat.

Experimental rigs for the investigation of convective heat transfer in the flows of various fluids were built at the Lithuanian Energy Institute. They enabled to measure the flow parameters in air, as well as in water and oil, using the same experimental channel. Two series of such measurements were carried out while investigating roughness influence. In the case of liquid flows, Pr from 0.71 to 100 were measured by Bartkus and results are presented in book [1]. The stress was laid on liquids. The turbulent flow structure and spatial temperature correlation also were measured. On the basis of these measurements Bartkus and Držus [1,2] have proposed a method for the theoretical calculation of the heat transfer by modifying the mixing length hypothesis near the wall.

Pedišius and Japertas [1] have investigated the effects of special turbulizers, suitable for air flow, on the heat transfer and drag. Attention was focused on the relationship between eddies created by roughness elements and those existing in the boundary layer. Inclined and indented ribs have been used. These measurements were continuation and verification of results published by Kalinin and others [3], who have shown that it is expedient to re-flare heat exchanger tubes forming ribs with rounded shapes. By suggestion of others it is advisable also to make cavities on surface at small Re.

This study had been done according to the material of experimental investigations published in [1]. Here two physically grounded rules are presented for new stress.

In the beginning, the necessary issues of general nature for this work must be emphasized. Hydraulic resistance of tubes with rough surfaces is presented in Fig. 1 as a fragment from the well-known diagram. On the right-hand side of the smooth surface turbulent friction curve there is the zone of transitional roughness,

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Nomenclature

c_f	friction coefficient $\tau_w/(\rho u_0^2/2)$
D	tube diameter (m)
f	friction factor for tubes $\Delta p/[(L/D)(\rho u_0^2/2)]$
h	height of roughness element (m)
k	hydraulic height of roughness (m)
k^+	roughness Reynolds number $u_* k/v$
L	length of the tube (m)
Δp	pressure drop (N/m^2)
Pr	Prandtl number
Re	Reynolds number
R_{tt}	correlation coefficient
r_z	transverse distance between thermoprobes (m)
s	step between roughness elements (m)
St	Stanton number
u	velocity (m/s)
u_*	friction velocity $(\tau_w/\rho)^{0.5}$

Greek symbols

α	heat transfer coefficient (W/m^2 K)
δ	boundary layer thickness (m)
ρ	density (kg/m^3)
v	kinematic viscosity (m^2/s)
τ_w	wall shear stress (N/m^2)

Subscript

o	smooth wall, mean flow quantity

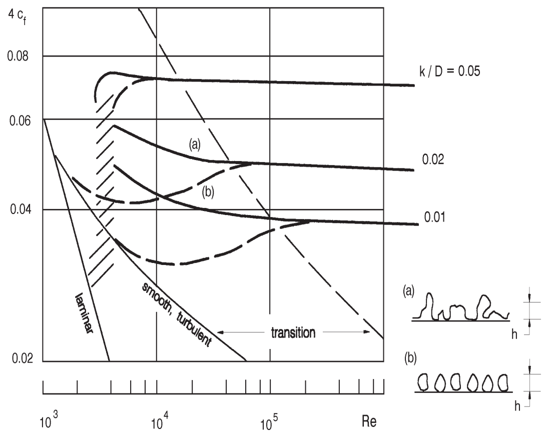

Fig. 1. The differences of friction in the transitional zone for roughness shapes corresponding (a) and (b).

where the flow depends on roughness height and Reynolds number. It borders on the zone of fully developed roughness, where the flow depends only on roughness height. Measurements by Nikuradze are shown by dotted lines. There, the roughness elements were formed by sand grains having the same diameter. The main salient features of these measurements are exact data on the hydraulic resistance and the roughness element's geometries. In this case, single-separate peaks were absent (Fig. 1(b)). The transition from the flow on a smooth surface to the entirely swirling flow is delayed. The curves are gradually declining from the curve of smooth surface. When stable eddies are developed the friction increases suddenly and square dependency on velocity establishes. Measurements made by Nikuradze are the foundation of theoretical understanding of the influence of the height of roughness elements on pressure losses, and it will be discussed in detail later.

Artificial roughness is created using elements of the same height and in this respect it is closely related to Nikuradze's experiments. It will be mentioned further, that elements having streamlined shape allow a delayed transition to fully developed flow. The curves remain little below Nikuradze's data.

In the zone of transitional roughness, the resistance of surfaces with elements having random form is shown by solid lines in Fig. 1. It corresponds to the widely used Moody diagram. Eddies arise at once behind single-element tops; therefore friction is greater and the variation law shows gradual eddy zones growing with the velocity. In the zone of fully developed roughness flow, the height of roughness elements is described as equivalent to grain roughness by comparing hydraulic resistance. Roughness height k is an average integral value corresponding to pressure drop resulting from the eddies and is named as hydraulic height.

For transitional region of roughness flow the friction is described by the formula of Colebrook–White:

$$\frac{1}{\sqrt{4c_f}} = 1.74 - 2\log\left(\frac{2k}{D} + \frac{18.7}{Re\sqrt{4c_f}}\right). \tag{1}$$

The values obtained using this formula are in accordance with Moody diagram.

Heat exchange of rough surfaces is more difficult to describe because of its additional dependency upon Pr. For fully developed rough case, it is possible to use universal functions for velocity and temperature distribution. For tubes with square repeated-rib roughness, Webb et al. [4] correlated heat transfer coefficients by the correlation which is good enough for use when $k^+ > 30$:

$$\frac{c_f}{2St} = 1 + \sqrt{\frac{c_f}{2}}\left(gPr^{0.43} - u_k\right), \tag{2}$$

$$g = 4.5(k^+)^{0.28}, \tag{3}$$

where u_k is Nikuradze's similarity function.

Universal formulas for the transitional regime do not exist as heat exchange depends on roughness shape and pitch differently at various Prandtl numbers. Usually, in each particular case, experimental results are described by empirical formulas such as in [3].

More sophisticated turbulent flow theories like k–ε model have included effects of roughness as wall functions. In brief manner, the fully developed roughness relationship is the basis for modelling. At this time theoretical proposals are not known to consider the new sensitive case of heat transfer enhancement at transition-flow regime for different roughness types at different Pr. Further information, that enables to perceive the case, will be provided in this work.

2. Rules and measurement results

Rule 1. For fluids with $Pr > 5$ it is always useful to design heat exchange surfaces with low-height roughness elements, uniformly and closely arranged.

The main resistance to heat exchange in the turbulent boundary layer of such fluids as cold water, glycerol mixtures, oil, etc. is the viscous sublayer. At $Pr = 5$ it causes 60% and at $Pr = 100$ it causes 95% of temperature drop. As eddies begin to form behind each roughness element, the sublayer becomes turbulised. Thermal sublayer thickness decreases and turbulisation influence increases with Pr.

It is well known that turbulisation becomes noticeable on heat transfer augmentation, when the roughness elements satisfy condition $k^+ = ku_*/v > 10$. To achieve its efficient influence, it is advantageous to use the transitional roughness flow regime, when k^+ is between 20 and 100. It provides sufficiently wide range for velocity variation when the process is controlled in heat exchange device. Friction velocity u_* and mean flow velocity u are linearly interrelated.

Parameter k^+ variation up to 100 is not arbitrary. If the streamlined form elements are used and they are arranged staggered with (2–30)k step distances, changes from transitional flow mode to fully rough case is delayed. Hydraulic drag coefficient values at $k^+ = 70$–100 are slightly less than Nikuradze's sand grain roughness. Small hydraulic drag increment is due to the height value of all elements being equal and due to the free space between them. Regime of transitional roughness flow past the bodies is favourable also because of its physical nature. In that case, the recirculation zone behind any roughness element is longer with few eddies and roughness elements can be sparsely arranged.

Now let us consider the heat intensification measurements on a flat plate carried out by Bartkus, which are described in the book [1]. After final generalisation the results are presented in Fig. 2. Comparison of these results among themselves are accurate, as experiments with three types of roughness chosen, represented in Fig. 3(a), were carried out in the same channel at the same velocities for each type of fluid. Results are shown in such a relative form that the efficiency of the rough surfaces are clearly seen. St_0 and c_{f0} correspond to turbulent flow on smooth surface. Main position of the first rule follows directly from results given in Fig. 2. At $Pr > 5$ efficiency exceeds 1 and at $Pr = 100$ value of 1.7 is reached. As it may be seen, the situation is possible, when Reynolds analogy is circumvented as the sublayer is destroyed by small eddies.

It is very important to pay due attention to the geometrical characteristics of the roughness. Three-dimensional uniformly spaced elements are preferable. This might be confirmed by data presented in Fig. 4, where comparison is made between results in oil, Pr being 102, and elements of different types: having pyramidal form and rectangular ribs without sharp edges arranged at small step $s/k = 3.2$. These ribs placed transversally to flow have a more expressed peak but are

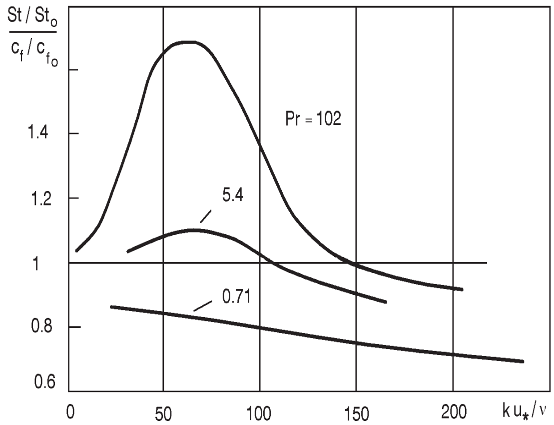

Fig. 2. Heat transfer efficiency at different Pr and k^+ for the pyramidal form of roughness elements shown in Fig. 3.

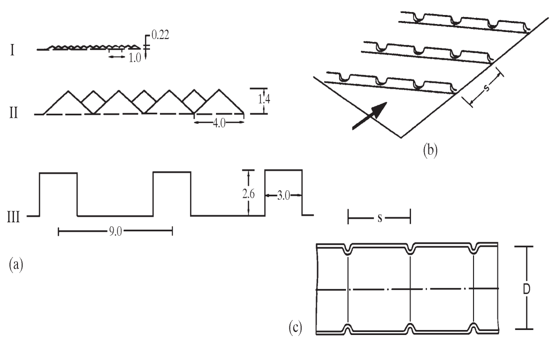

Fig. 3. The type of roughness considered in this work: (a) – small height elements used in liquids, Bartkus [2], (b) – example of structural roughness used in air, Japertas [1] and (c) – spiral pressing in tube, Kalinin [3].

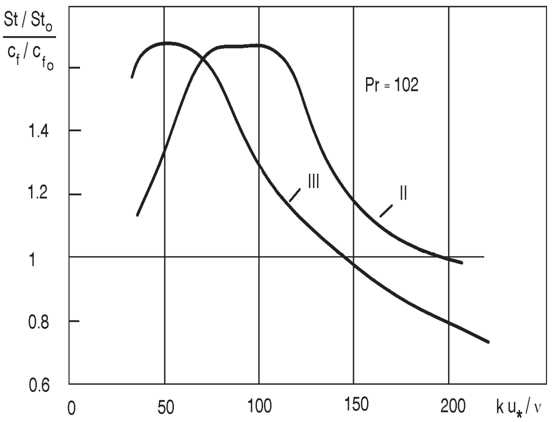

Fig. 4. Differences in effectiveness for two types of roughness at higher Pr: II – pyramidal three-dimensional form, III – transverse rectangular ribs according to Fig. 3.

not so useful in wider range of velocities. Rounded three-dimensional elements generate more expressed and spread along flow, attached to the wall eddies, and the transition to the fully rough case is delayed. Air turbulisation by small size roughness elements is too low and ineffective.

All these results at various Re are valid for the pure surfaces, free of sediments. It is not advisable to plan the heat transfer improvement for more than three times. To design the useful heat exchange surfaces, when the liquid has $Pr > 5$, the choice of parameter k^+ is necessary. The Moody diagram of hydraulic drag coefficient of tubes is most suitable for this purpose. The relation between these parameter and diagram variables is

$$k^+ = \frac{k}{D} Re \sqrt{\frac{f}{8}}. \tag{4}$$

The height of the roughness elements is not necessary always identical to the hydraulic height k and determination of this value will be discussed later. It will be recalled that the link between hydraulic drag coefficient f for tube and friction factor of a plate in free stream is

$$c_f/2 = f/8, \tag{5}$$

while the Re are determined differently in these two cases. The mean cross-sectional velocity and the pipe diameter are used for Re calculation, while for freely developing flow the boundary layer thickness and maximum velocity is preferable. The numerical values are different, as the velocity in the pipe Re is 0.85 times less, while the pipe diameter is approximately two times larger than boundary layer thickness.

Rule 2. Two- to three-fold enhancement of heat transfer in gases can be reached by means of ordered turbulisation elements on the wall, and this rule may be usefully applied until the surface drag has not increased more than four times.

The detailed observation of flow past the separate roughness element leads to the understanding, that the main heat transfer intensification is due to the impact of new colder masses on the surface with the turbulised sublayer becoming thinner. Flow reattachment behind the obstacle on the surface has been widely investigated by many researchers working in the field of fluid dynamics, while description of heat transfer variation in this case can be found together with local heat transfer coefficient measurements in [1]. The more intensively the layers are turned over, mixed and flow dividing points are expressed, the bigger heat transfer increasing rates and effects on the air flow we have, because there is a thick layer with almost constant temperature. So changes from liquids with great Pr to gas flows require an increase in the dimensions of the roughness elements.

Detailed heat transfer measurement results are presented in Fig. 5 on the surface between roughness elements for two cases, when slightly-rounded ribs were placed on the surface spaced in steps $s/k = 8$ and $s/k = 14$. It can be seen that for roughness with smaller step the flow is being detached at the obstacle itself and the heat transfer coefficient varies monotonically from one rib to the another. The minimal value is always behind the obstacle. If the distance between elements is increased, the stream impact will become stronger. The maximum value of the heat transfer increases with the eddy before the obstacle creating a secondary maximum. In this case, the averaged heat transfer coefficient is larger.

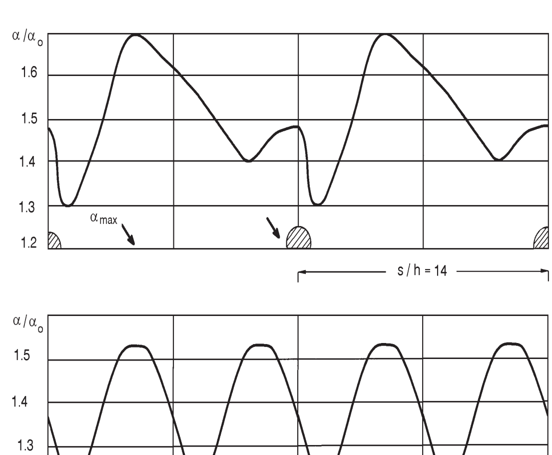

Fig. 5. Measured local heat transfer coefficient in between the roughness elements at two different steps. α_o – heat transfer on smooth wall.

To summarise briefly, the roughness elements are creating heat transfer coefficient peak values by the flow reattachment and setting up of the flow dividing point. Physical foundation of heat transfer intensification by the use of boundary layer origin is close to the utilisation of the assembly of shot fins when advantage is taken to the greatest heat exchange on the front edge of the fin, where the boundary layer begins to form.

There is an additional mechanism by which the roughness is influencing the improvement of the heat transfer across the boundary layer: the strengthening of secondary eddies in the flow by special set of roughness. It is possible to arrange the roughness closely acting on the large eddies intensity in the boundary layer and turn over its fluid masses. In the boundary layer, transfer is been determined by a sweep of faster masses towards the wall. Correlation measurements of coherent eddy structures show the exchange intensification across the boundary layer for the rough surface case, Fig. 6. Due to rough surface retardation the probability of sweep appearance increases four times compared to smooth surface. This phenomenon may draw attention to the fact that movement of masses perpendicular to the wall is useful in case of air flow. Intensity of lateral movements presented in Fig. 6 increases when roughness elements initiate turn over of masses having frequency typical to the eddies of the boundary layer. The best case is when the roughness elements have streamlined form and therefore do not have own distinct frequency and do not have additional waste of flow energy. It is purposive to name this specific roughness by the separate name-structural roughness.

Another type of secondary eddies initiation is expedient in the practice. It is been generally known type of eddies due to sharp channel angles. Such eddies may be induced everywhere using duly arranged wall turbulizers of due form. Influence of this secondary flow in channel on heat transfer was been registered in our investigation by placing the ribs oblique to flow.

Heat transfer values when the useful roughness was been used, have been measured on the plate and in the channel. Measurements in the channel gave accurate data about friction. The one type of roughness used is shown in Fig. 3(b).

The study by Pedišius [1] is related to defining the type of structural roughness. Intensification was achieved using lateral ribs, perpendicular to flow, oblique ribs, forming 45° angle and ribs with discontinuities. It became certain that the best choice of obstacle type is such, when roughness is delaying transition to the fully developed rough case. The best is oblique positioned to flow ribs.

Results of these measurements in generalised form are presented in Fig. 7. Assertion of Rule 2 follows immediately from these results. Utilisation of oblique rib is the most effective means followed by rounded ribs perpendicular to flow direction or less rounded with discontinuities, cut outs. It should be noted that optimum value has reached when k^+ approaches 100.

Effectiveness of geometrical parameters of ribs in the transitional regime may be stated in following sequence: ribs height and its angle to flow direction, then ribs spacing and their form. The angle of mounted ribs with respect to flow direction determines formation of secondary eddies and streams along ribs. Optimal spacing

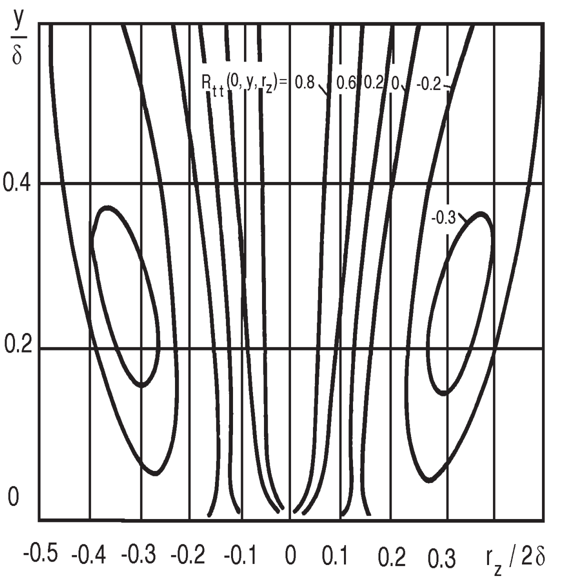

Fig. 6. Spatial correlation of temperature fluctuations in air flow on rough surface across the boundary layer.

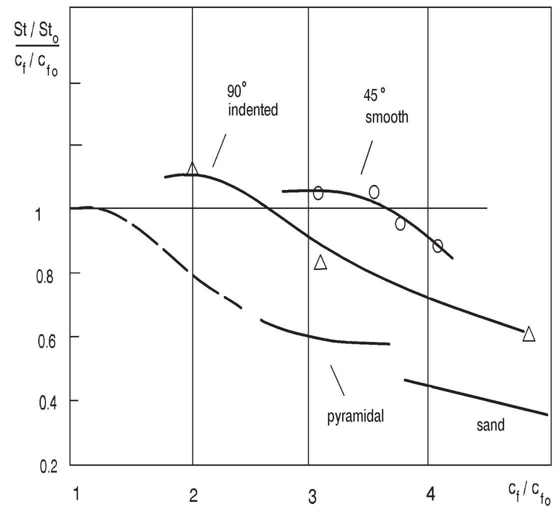

Fig. 7. Effectiveness of various roughness geometries in air flow vs its friction.

step is 14 for rectangular and 16 for half-cylinder ribs, bigger spacing are better than smaller ones.

Such surfaces for air or other gas flows may be designed using data presented here for structural roughness. Hydraulic resistance diagram for pipe is useful. If the turbulent flow on the smooth surface case is known, then the friction factor must be increased according to Rule 2 three to four times and the height of roughness elements must be determined. Ribs are designed according to this height and set oblique having some angle in respect to flow axis.

3. Roughness geometry

In our methodology, it is necessary to use such dimension as roughness element height in absolute measure units, that is their height in millimetres. For technical purposes, the friction curve $c_{\mathrm{f}} = f(Re, k/D)$ has been drawn by using relative roughness element height. Let us find the relation between geometrical element height and hydraulic height according to this relationship. The hydraulic roughness height is directly linked with not only the height of elements but also with their arrangement. The resistance is the same for all hydraulic heights when diameters and Re are the same. This is the case, when elements, although different in form, retard the flow in the same way.

Let us assume that the retardation of the separate element is proportional to the volume of recirculating eddy zones behind the element. Verification of this assumption was made in accordance with the measurements carried out by Schlichting [5]. The case of spheres placed on surface was considered. Their arrangement, diameters of spheres and uniform-grain roughness k found by hydraulic measurements are shown in Table 1. The calculated k values are given in the last row.

The calculation has been carried out in the following manner. Identity of volume turbulised by elements was assumed according to idea described. First, volume of recirculation zone of the individual element height h was found which has supposed to take up $20h$ behind the element. In this case, the volume full of eddies behind the element of spherical form (approximately rectangular) is $V = 20h^3$. At the same time it should be noted that the chosen single element has some surface area about it as a house has its plot of ground around it. This surface must be fully covered by vortical flow created by sand-grain roughness with height k. In this case, the volume full of eddies is $V = ks^2$, where s is the elements displacement pitch at a staggered displacement of elements according to analysing case [5]. The balance of volumes full of eddies inside may be written and hydraulic height found

$$k = 20h^3/s^2. \tag{6}$$

It can be seen from Table 1 that values of measured and calculated k are in good agreement. Hence the rule we need may be deduced: having uniform-grain roughness k obtained from diagram $c_{\mathrm{f}}(Re, k/D)$ can be designed elements of desired form and desired arrangement. Recalculation factor 20 is universal enough for elements of streamlined form having several millimetres in height. This factor, if calculated from data of the same literature, for elements more flattened to the wall, semispheres and cones, is equal to 25. Above mentioned method cannot be used for elements having sharp edges, causing a wide expansion of recirculation zone.

There is another hint from the measurement analysis aforementioned as regards to the determination of k value for continuous ribs. Assuming that the length of turbulised zone equals to $20h$, and finding that this factor fits to measurements the circulation zone of continuous ribs ought not to be less then this value. Hence, if ribs are placed at spacing steps less then $20h$, value k is always equal to geometric rib height h. It is interesting that in practice this spacing step $20h$ is the most effective one. It will be recalled that hydraulic height k for closely spaced grain roughness elements is less then that of the geometric height of elements, for example, in case of close positioned spheres, it is less by one third [5].

4. Conclusions

In this paper, attention is stressed on the effective heat transfer enhancement by roughness of small height in the region, where it is analog with hydraulic resistance augmentation. On physical grounds two rules have been derived, one for liquids and another for gases. This knowledge was been drawn from our measurement results in liquids with different Pr at various roughness geometry. In air flow, the more physical insight of phenomena was used and the investigation of more

Table 1
Measured and calculated roughness hydraulic height k

Case of roughness (mm)	case 1	case 2	case 3	case 4	case 5	case 6
Element height h	4.1	4.1	4.1	4.1	2.1	2.1
Step s	40	20	10	0	10	5
Measured k [5]	0.93	3.4	12.6	2.57	1.72	7.59
Calculated k	0.86	3.4	13.8	–	1.85	7.41

effective roughness geometry was done. For gaining the information the measurements of local heat transfer coefficient, flow structure and the space correlation of temperature fluctuations were been included.

In the present study, we can summarize the following:

1. For turbulent flow case, it is possible to improve heat transfer up to 2–3 times at *Re* numbers reaching 10^5 using roughness with small friction. The use of the ribs with slightly rounded corners and oblique to flow or three-dimensional obstacles are delaying the transition from smooth wall case to the fully developed roughness.
2. Heat exchange effectiveness $(St/c_\mathrm{f})/(St/c_\mathrm{f})_0$ of liquids with $Pr > 5$ exceeds Reynolds analogy and has its maximum at $ku^*/v = 50$. Small equally dispersed roughness elements having three-dimensional form are eddy generators good enough to turbulise viscous sublayer.
3. For air flow case, roughness elements of large scale are more suitable. Here heat exchange increase is been determined by flow reattachment and mixing of complete boundary layer by, for example, means of secondary eddies. With elements of special form it is possible to delay transition and work at $ku^*/v = 100$.

By choosing roughness height and form it is necessary to follow these two rules mentioned above. Additionally in this work, it has been shown how to arrange preferable roughness suitable for the desired case. The first approach to compare different roughness is the use of equal volumes of recirculating vortex flow behind roughness elements. Example of such a calculation has been done according to measurements of Schlichting. This idea allows us to design the roughness geometry by using the Moody diagram.

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